Heat Kernel and Scaling of Gravitational Constants

Diego A. R. Dalvit

Departamento de Física, Facultad de Ciencias Exactas y Naturales

Universidad de Buenos Aires- Ciudad Universitaria, Pabellón I

1428 Buenos Aires, Argentina

Francisco D. Mazzitelli

Departamento de Física, Facultad de Ciencias Exactas y Naturales

Universidad de Buenos Aires- Ciudad Universitaria, Pabellón I

1428 Buenos Aires, Argentina

and

Instituto de Astronomía y Física del Espacio

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Abstract

We consider the non-local energy-momentum tensor of quantum scalar and spinor fields in $2w$-dimensional curved spaces. Working to lowest order in the curvature we show that, while the non-local terms proportional to $\Box\mathcal{R}$, $\Box\Box\mathcal{R}$, $\dots$, $\Box^{w-2}\mathcal{R}$ are fully determined by the early-time behaviour of the heat kernel, the terms proportional to $\mathcal{R}$ depend on the asymptotic late-time behaviour. This fact explains a discrepancy between the running of the Newton constant dictated by the RG equations and the quantum corrections to the Newtonian potential.

I. INTRODUCTION

In a recent paper [1] we have computed the corrections to the Newtonian potential due to a quantum massive scalar field coupled to the metric in a $R+R^2$-theory of gravitation. This computation was carried out by means of a non-local approximation to the Effective Action (EA) [2,3], from which the effective gravitational equations of motion were deduced. Expanding in powers of $-\frac{m^2}{\Box}$, these equations read

$$\left[\alpha_0 - \frac{1}{64\pi^2}\left((\xi-\frac{1}{6})^2-\frac{1}{90}\right)\ln(-\frac{\Box}{\mu^2})\right]H^{(1)}_{\mu\nu} + \left[\beta_0 - \frac{1}{1920\pi^2}\ln(-\frac{\Box}{\mu^2})\right]H^{(2)}_{\mu\nu} +$$
$$\left[-\frac{1}{8\pi G}+\frac{m^2}{16\pi^2}(\xi-\frac{1}{6})(-1+\ln\frac{m^2}{\mu^2})\right]\left(R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}\right) -$$
$$\frac{m^2}{384\pi^2}\ln(-\frac{\Box}{m^2})\frac{1}{\Box}\left[(1-12\xi^2)H^{(1)}_{\mu\nu}-2H^{(2)}_{\mu\nu}\right] = O(\mathcal{R}^2) \tag{1}$$

where m is the mass of the scalar field, ξ is the coupling to the scalar curvature and

$$H^{(1)}_{\mu\nu} = 4\nabla_\mu\nabla_\nu R - 4g_{\mu\nu}\Box R + O(\mathcal{R}^2)$$
$$H^{(2)}_{\mu\nu} = 2\nabla_\mu\nabla_\nu R - g_{\mu\nu}\Box R - 2\Box R_{\mu\nu} + O(\mathcal{R}^2) \tag{2}$$

The gravitational constants α_0, β_0 and G depend on the scale μ according with the Renormalization Group Equations (RGEs) [4]

$$\mu\frac{d\alpha_0}{d\mu} = -\frac{1}{32\pi^2}\left[\left(\xi-\frac{1}{6}\right)^2-\frac{1}{90}\right] \tag{3}$$
$$\mu\frac{d\beta_0}{d\mu} = -\frac{1}{960\pi^2} \tag{4}$$
$$\mu\frac{dG}{d\mu} = \frac{G^2m^2}{\pi}\left(\xi-\frac{1}{6}\right) \tag{5}$$

These are basically given by the Schwinger-DeWitt (SDW) coefficients and can be obtained by imposing Eqn.(1) to be independent of the renormalization scale μ. Comparing the RGEs with the effective Eqn.(1) one readily notes that, while the corrections proportional to $\ln(-\frac{\Box}{\mu^2})$ can be interpreted as non-local modifications to α_0 and β_0, this is not the case for the Newton constant. Indeed, because of the identity

$$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu} = \frac{1}{4\Box}(H^{(1)}_{\mu\nu}-2H^{(2)}_{\mu\nu}) + O(\mathcal{R}^2) \tag{6}$$

the non-analytic corrections proportional to $-\frac{m^2}{\Box}\ln(-\frac{\Box}{m^2})$ can be interpreted as modifying G only for $\xi = 0$. This has also been pointed out in Ref. [5].

This discrepancy can also be seen at the level of the Newtonian potential, which has $\frac{\log r}{r}$ and r^{-3} quantum corrections [1]. The r^{-3} corrections come from the $\ln(\frac{-\Box}{\mu^2})$ terms in the effective equations and survive in the massless limit (similar corrections due to the graviton sector of the theory have been found in [6]). The $\frac{\log r}{r}$ corrections come from the term proportional to $-\frac{m^2}{\Box}\ln(-\frac{\Box}{m^2})$. In principle, one could 'derive' these logarithmic corrections from the RGE (5), replacing in the classical potential $V_{cl}(r)$ the Newtonian constant by its running counterpart and identifying $\mu \leftrightarrow r^{-1}$. The resulting 'Wilsonian' potential $V(r) = -G(\mu = r^{-1})/r$ coincides with the one obtained in Ref. [1] only for minimal and conformal coupling. [1]

The aim of this work is to elucidate the origin of the discrepancy between the scaling behaviour of the Newton constant deduced from the effective equations of motion and that obtained through the RGEs. To this end we will show that there is a qualitative difference between the non-local corrections proportional to $\ln(-\Box)$ and those proportional to $-\frac{m^2}{\Box}\ln(-\Box)$. While the former are linked to the *early-time* behaviour of the heat kernel [7] (and consequently are determined by the $\hat{a}_2$ SDW coefficient), the latter depend on the *late-time* behaviour and produce the above-mentioned discrepancy. We will prove this claim in Section II, where we will also extend the four-dimensional results to arbitrary dimensions. In Section III we will analyze the same problem for spinor fields.

We emphasize that throughout this paper we will consider quantum matter fields on a classical gravitational background. This will be enough for our main discussion, since at this *semiclassical* level we already have running coupling constants and quantum corrections to the field equations and Newtonian potential. Therefore we can compare both answers and look for the reason of the discrepancy.

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[1] The coincidence at $\xi = 1/6$ takes place only after tracing the equations of motion.

In order to go beyond the semiclassical theory, there are two alternatives. If the $R+R^2$-theory is considered as an effective, low-energy field theory [8,9], the inclusion of the graviton sector can be done along the lines of Ref. [6], and we expect additional r^{-3} corrections to the Newtonian potential. On the other hand, if the $R+R^2$-theory is considered as a complete and renormalizable theory of gravity, due to asymptotic freedom [10], the graviton sector could produce an important increase of G with distance [11]. However, in this case the $R+R^2$-theory is non-unitary, and no definite conclusions can be drawn. This point is beyond the scope of this paper.

II. SCALING FOR SCALAR FIELDS

Let us consider the evaluation of the one-loop contribution of a massive quantum scalar field to the gravitational EA

$$\Gamma = \frac{1}{2}\ln\det(-\Box + m^2 + \xi R) \tag{7}$$

The task of evaluating this functional determinant on an arbitrary background is quite complicated and approximation methods are compelling. Using the early-time expansion of the heat kernel, the EA in $2w$ dimensions reads [2,3,7]

$$\Gamma = -\frac{1}{2}\lim_{L^2\to\infty}\frac{1}{(4\pi)^w}\int_{1/L^2}^{\infty}\frac{ds}{s^{w+1}}\exp(-sm^2)\sum_{n=0}^{\infty}s^n\int d^{2w}x\sqrt{g}\,\hat{a}_n(x) \tag{8}$$

where the ultraviolet divergence is regularized by the introduction of a positive lower limit in the proper-time integral. Here all the functions $\hat{a}_n(x)$ are the coincident limit of the SDW coefficients.

As suggested by Vilkovisky [7] , when the background fields are weak but rapidly varying, one can obtain a non-local expansion of the EA by summing all terms with a given power of the curvature and any number of derivatives in the SDW series. The result is well-behaved in the massless limit and can be written as

$$\Gamma = -\frac{1}{2}\frac{1}{(4\pi)^w}\int d^{2w}x\sqrt{g}\lim_{L^2\to\infty}\left(h_0 + h_1(\frac{1}{6}-\xi)R + RF_1(\Box)R + R_{\mu\nu}F_2(\Box)R_{\mu\nu} + O(\mathcal{R}^3)\right) \tag{9}$$

where $h_n = \int_{1/L^2}^{\infty} ds s^{n-w-1} e^{-sm^2}$, $F_i(\Box) = \int_{1/L^2}^{\infty} ds \frac{e^{-sm^2}}{s^{w-1}} f_i(-s\Box)$ and the form factors f_i are functions to be defined afterwards.

Up to here no assumptions about the mass m have been made. In the large mass limit, $m^2\mathcal{R} \gg \nabla\nabla\mathcal{R}$, the SDW expansion is recovered, while in the opposite one, the form factors can be expanded in powers of $z \equiv -\frac{m^2}{\Box}$. We shall be working in the latter limit. We have to evaluate the integral

$$I_w \stackrel{\text{def}}{=} \lim_{L^2\to\infty} \int_{1/L^2}^{\infty} ds \frac{e^{-m^2 s}}{s^{w-1}} \sigma(-s\Box) \tag{10}$$

where σ denotes generically the f_i's. In order to study the behaviour of I_w in terms of the small quantity z, we split up the integral into two terms

$$\begin{aligned} I_w &= \lim_{L^2\to\infty} (A_w + B_w) \\ A_w &= (-\Box)^{w-2} \int_{-\Box/L^2}^{C} \frac{d\eta}{\eta^{w-1}} e^{-\eta z} \sigma(\eta) \\ B_w &= (-\Box)^{w-2} \int_{C}^{\infty} \frac{d\eta}{\eta^{w-1}} e^{-\eta z} \sigma(\eta) \end{aligned} \tag{11}$$

where C is chosen such that $z^{-1} \gg C \gg 1$. Let us analyze the two integrals separately.

For the A_w integral, one can use the Taylor expansion of the form factor, namely $\sigma(\eta) = \sum_{n=2}^{\infty} \sigma_n \eta^{n-2}$. The constants σ_n can be read from the corresponding SDW coefficient $\hat{a}_n$, as follows from Eqns(8,9). The $n \geq w+1$ terms have a finite $L^2 \to \infty$ limit that gives a $\Box$-dependent contribution that is analytic in the variable z, while the $2 \leq n \leq w$ terms are UV divergent. Expanding the exponential in A_w in powers of the small quantity ηz we obtain its final expression

$$\begin{aligned} A_w = &-(-\Box)^{w-2} Log(-\frac{\Box}{L^2}) \sum_{n=2}^{w} \frac{\sigma_n}{(w-n)!} (-\frac{m^2}{\Box})^{w-n} + \\ &(-\Box)^{w-2} \sum_{n=2}^{w} \sum_{k=0}^{w-n-1} \frac{\sigma_n}{(w-n-k)k!} (\frac{m^2}{\Box})^2 (-\frac{L^2}{\Box})^{w-n-k} + \dots \end{aligned} \tag{12}$$

where the dots denote finite terms, analytic in the small quantity $-\frac{m^2}{\Box}$. Note that both the divergent and non-analytic parts of A_w are determined by the first w SDW coefficients. In order to renormalize the theory, the infinities have to be cancelled by means of suitable

counterterms in the classical lagrangian of the form $\mathcal{R}\mathcal{R}$, $\mathcal{R}\Box\mathcal{R}$, $\mathcal{R}\Box^2\mathcal{R}, \ldots, \mathcal{R}\Box^{w-2}\mathcal{R}$, these being the only quadratic counterterms that can appear. The UV divergences proportional to $\ln(L^2)$ that appear in both A_w and the h_n integrals are absorbed in the bare constants, being renormalized by terms of the form $\log(\frac{L^2}{\mu^2})$, where μ is an arbitrary arbitrary scale parameter with units of mass. The fact that the EA must not depend on this arbitrary parameter implies that the gravitational constants scale with μ, the scaling being given by the RGEs (see Eqns(3,4,5) for the $w = 2$ case).

As to the B_w integral, its leading behaviour in powers of $-\frac{m^2}{\Box}$ is governed by the asymptotic expansion of the form factor. Assuming that $\sigma(\eta) = \frac{k}{\eta}$ as $\eta \to \infty$, where k is a numerical factor, the integral B_w reads

$$B_w = k\frac{(-1)^w}{(w-1)!}(-\Box)^{w-2}(-\frac{m^2}{\Box})^{w-1}\ln(-\frac{m^2}{\Box}) + \ldots \tag{13}$$

the dots being analytic terms.

Given the EA one can derive the effective gravitational field equations. After a straightforward calculation we find

$$\left(-\frac{1}{8\pi G} + \frac{(-1)^w(m^2)^{w-1}}{(4\pi)^w(w-1)!}(\xi - \frac{1}{6})\ln(\frac{m^2}{\mu^2})\right)(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}) + \sum_{j=0}^{w-2}\left[\alpha_j\Box^j H^{(1)}_{\mu\nu} + \beta_j\Box^j H^{(2)}_{\mu\nu}\right] =$$
$$< T_{\mu\nu} > \stackrel{\text{def}}{=} -\frac{1}{2(4\pi)^w}[F_1(\Box)H^{(1)}_{\mu\nu} + F_2(\Box)H^{(2)}_{\mu\nu}] + O(\mathcal{R}^2) \tag{14}$$

In this equation the cosmological constant term has been omitted and α_j and β_j denote the gravitational constants associated with the higher order terms in the classical lagrangian.

In four-dimensional spacetime the basic integral I_w can be calculated using Eqns(12,13). Up to analytic terms in $-\frac{m^2}{\Box}$ it is given by

$$I_{w=2} = -\sigma_2 Log(-\frac{\Box}{\mu^2}) - k\frac{m^2}{\Box}Log(-\frac{m^2}{\Box}) + O\left(-\frac{m^2}{\Box}\right)^2 \tag{15}$$

The corresponding stress tensor reads

$$< T_{\mu\nu} > = \frac{1}{32\pi^2}\left(\log(-\frac{\Box}{\mu^2})[\sigma_2^{(1)}H^{(1)}_{\mu\nu} + \sigma_2^{(2)}H^{(2)}_{\mu\nu}] + \frac{m^2}{\Box}\log(-\frac{m^2}{\Box})[k^{(1)}H^{(1)}_{\mu\nu} + k^{(2)}H^{(2)}_{\mu\nu}]\right) \tag{16}$$

the $\sigma_2^{(i)}$ and $k^{(i)}$ being the numerical constants in Eqn(15), respectively associated with the R^2 and $R_{\mu\nu}R_{\mu\nu}$ terms in the EA.

The m^2-independent terms in $< T_{\mu\nu} >$ can be interpreted as being quantum corrections to the gravitational constants α_0 and β_0. As was already mentioned, the numerical coefficients $\sigma_2^{(i)}$ associated with these corrections are basically given by the $\hat{a}_2$ SDW coefficient (early-time behaviour of the heat kernel). When the equations of motion are traced and solved, these terms produce r^{-3} quantum corrections to the Newtonian potential [1].

In an analogous way, one would expect that the m^2-dependent terms in $< T_{\mu\nu} >$, namely

$$\frac{m^2 k^{(1)}}{32\pi^2}\log(-\frac{m^2}{\Box})\frac{1}{\Box}(H^{(1)}_{\mu\nu} + \frac{k^{(2)}}{k^{(1)}}H^{(2)}_{\mu\nu}) \tag{17}$$

could be expressed in a combination proportional to $m^2\log(-\frac{\Box}{m^2})(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu})$, so that they can be interpreted as a quantum correction to the Newton constant. From Eqn(6) we see that the aforementioned combination comes up only for $k^{(2)}/k^{(1)} = -2$, a condition that is not always met. Also note that the correction depends on the numerical coefficients $k^{(i)}$, which are given by the asymptotic late-time behaviour of the heat kernel. The terms in Eqn(17) produce a $\frac{\log r}{r}$ correction to the Newtonian potential [1].

The coefficients σ_n's and k's can be evaluated from the form factors f_i's. These are defined through the basic form factor $f(\eta) = \int_0^1 dte^{-t(1-t)\eta}$ as follows [3,12]

$$\begin{aligned} f_1(\eta) &= \frac{f(\eta)}{8}\left[\frac{1}{36} + \frac{1}{3\eta} - \frac{1}{\eta^2}\right] - \frac{1}{16\eta} + \frac{1}{8\eta^2} + (\xi - \frac{1}{6})\left[\frac{f(\eta)}{12} + \frac{f(\eta)-1}{2\eta}\right] + \frac{1}{2}(\xi - \frac{1}{6})^2 f(\eta) \\ f_2(\eta) &= \frac{f(\eta) - 1 + \eta/6}{\eta^2} \end{aligned} \tag{18}$$

From here the relevant coefficients for the four-dimensional theory can be calculated: $\sigma_2^{(i)} = f_i(0)$ and $k^{(i)} = \lim_{\eta\to\infty}\eta f_i(\eta)$. Therefore we have

$$\begin{aligned} \sigma_2^{(1)} &= \frac{1}{2}\left[\left(\frac{1}{6} - \xi\right)^2 - \frac{1}{90}\right] \qquad \sigma_2^{(2)} = \frac{1}{60} \\ k^{(1)} &= \xi^2 - \frac{1}{12} \qquad k^{(2)} = \frac{1}{6} \end{aligned} \tag{19}$$

It is straightforward to see that only for minimal coupling ($\xi = 0$) can the m^2-dependent part of $< T_{\mu\nu} >$ be interpreted as correcting the Newton constant.

All this reasoning case be extended for arbitrary values of w. All terms in the energy-momentum tensor that depend on the σ_n's can be interpreted as being quantum corrections to the gravitational constants associated with the corresponding $\mathcal{R}\Box^{n-2}\mathcal{R}, (2 \leq n \leq w)$ terms in the classical lagrangian. The numerical coefficients σ_n's in these corrections depend on the $\hat{a}_n$ SDW coefficient. On the contrary, the terms with higher power of the mass (k-dependent ones) involve the asymptotic behaviour of the non-local form factors and can be viewed as correcting the Newton constant only for $\xi = 0$.

For example, in six-dimensional spacetime the integral I_w can be calculated using Eqns(12,13) and is given by

$$I_{w=3} = \sigma_3 \Box Log(-\frac{\Box}{\mu^2}) + \sigma_2 m^2 Log(-\frac{\Box}{\mu^2}) - k\frac{m^4}{2\Box} Log(-\frac{\Box}{m^2}) \tag{20}$$

For this theory the coefficients σ_2 and k are the same as those of the four-dimensional one, while the σ_3 coefficients are obtained from the term of the form factors that is linear in η and read

$$\sigma_3^{(1)} = -\frac{1}{336} + \frac{\xi}{30} - \frac{\xi^2}{12} \qquad \sigma_3^{(2)} = -\frac{1}{840} \tag{21}$$

In this case one obtains that the m^0 (m^2) terms in $< T_{\mu\nu} >$ are interpreted as quantum corrections to the gravitational coefficients α_0, β_0 (α_1, β_1) and depend on the $\hat{a}_2$ ($\hat{a}_3$) SDW coefficient. As before, one can view the m^4 terms as a quantum correction to the Newton constant only for minimal coupling.

Having evaluated the energy-momentum tensor, we shall make a brief comment on the trace anomaly. As is well-known [4], the classical theory is conformally invariant for $m = 0$ and $\xi = \frac{1}{4}\frac{2w-2}{2w-1}$. Due to quantum effects, a trace anomaly in $< T_\mu{}^\mu >$ appears, which is local and proportional to the $\hat{a}_w$ SDW coefficient. In our computation of the energy-momentum tensor we have concentrated on the non-local terms and we have absorbed the local ones into the renormalized classical gravitational constants. Using the expressions for the coefficients $\sigma_w^{(i)}$ evaluated at conformal coupling (see Eqns(19,21) for the $w = 2$ and $w = 3$ cases) one can readily prove that the trace of the non-local and mass-independent terms of the energy-momentum tensor vanishes. Although the local terms are irrelevant for the main point of

this work, which is throughly developed in previous paragraphs, their evaluation from the integral A_w is straightforward. At conformal coupling these terms give the correct trace anomaly, up to the order we are working (the $O(\mathcal{R}^2)$ contributions to the trace anomaly have been recently computed from the non-local effective action in Ref. [13])

III. SCALING FOR SPINOR FIELDS

In this section we shall extend the reasoning to spinor fields in four dimensions. The one-loop contribution to EA of the free Dirac field on a gravitational background is

$$\Gamma = -\tfrac{1}{2}\mathrm{Tr}\ln\hat{K}$$
$$\hat{K}\Psi = (\gamma_\mu\nabla_\mu + m)(-\gamma_\nu\nabla_\nu + m)\Psi = (-\Box + m^2 + \frac{1}{4}R)\Psi \tag{22}$$

Therefore we have to evaluate the trace of an operator similar to that associated with the scalar field for $\xi = 1/4$ and trace over the spinor indexes.

We shall evaluate the EA following the method described in the previous Section (see Eqn(9)). The second order term in curvatures can be written as [2,3]

$$\Gamma^{(2)} = \frac{1}{32\pi^2}\int d^4x\sqrt{g}\,[4RF_1(\Box)R + 4R_{\mu\nu}F_2(\Box)R_{\mu\nu} + \mathrm{Tr}(\mathcal{R}_{\mu\nu}F_3(\Box)\mathcal{R}_{\mu\nu})] \tag{23}$$

where $\mathcal{R}_{\mu\nu} = [\nabla_\mu, \nabla_\nu] = \frac{1}{8}[\gamma_\alpha(x), \gamma_\beta(x)]_- R_{\alpha\beta\mu\nu}(x)$ is the commutator of the covariant derivatives [14]. Here $F_1(\Box)$ and $F_2(\Box)$ are the scalar field-form factor integrals evaluated at $\xi = 1/4$. We have a new contribution proportional to $F_3(\Box) = \int_{1/L^2}^{\infty} ds \frac{e^{-sm^2}}{s^{w-1}}\frac{f(-s\Box)-1}{2s\Box}$, due to the non-vanishing commutator of the covariant derivatives.

Using the expression for $\mathcal{R}_{\mu\nu}$ and calculating the trace of the product of four gamma matrices, the last term in Eqn(23) can be written as $\mathrm{Tr}\mathcal{R}_{\mu\nu}F_3(\Box)\mathcal{R}_{\mu\nu} = -\frac{1}{2}R_{\alpha\beta\mu\nu}F_3(\Box)R_{\alpha\beta\mu\nu}$. Finally, using integration by parts, the Bianchi identities and the non-local expansion of the Riemann tensor in terms of the Ricci tensor [3,12]

$$R_{\alpha\beta\mu\nu} = \frac{1}{\Box}\{\nabla_\mu\nabla_\alpha R_{\nu\beta} + \nabla_\nu\nabla_\beta R_{\mu\alpha} - \nabla_\nu\nabla_\alpha R_{\mu\beta} - \nabla_\mu\nabla_\beta R_{\nu\alpha}\} + O(\mathcal{R}^2)\ , \tag{24}$$

one can rewrite the last expression through a kind of generalized Gauss-Bonnet identity, namely

$$\int d^4x \mathrm{Tr}\mathcal{R}_{\mu\nu} F_3(\Box)\mathcal{R}_{\mu\nu} = \int d^4x \left[\frac{1}{2} R F_3(\Box) R - 2R_{\mu\nu} F_3(\Box) R_{\mu\nu} + O(\mathcal{R}^3)\right] \tag{25}$$

In view of this identity, the stress tensor is basically the one for the scalar field, modified as follows

$$\begin{aligned} < T_{\mu\nu} >= -\frac{1}{32\pi^2}\Bigg(&\log(-\frac{\Box}{\mu^2})\left[(4\sigma_2^{(1)} + \frac{1}{2}\sigma_2^{(3)})H^{(1)}_{\mu\nu} + (4\sigma_2^{(2)} - 2\sigma_2^{(3)})H^{(2)}_{\mu\nu}\right] + \\ &\frac{m^2}{\Box}\log(-\frac{m^2}{\Box})\left[(4k^{(1)} + \frac{1}{2}k^{(3)})H^{(1)}_{\mu\nu} + (4k^{(2)} - 2k^{(3)})H^{(2)}_{\mu\nu}\right]\Bigg) \end{aligned} \tag{26}$$

The new coefficients, associated to the form factor integral F_3, are given by $\sigma_2^{(3)} = 1/12$ (early-time behaviour) and $k^{(3)} = 1/2$ (late-time behaviour), and the other coefficients, written in Eqn(19), are evaluated at $\xi = 1/4$. Therefore the m^2-dependent terms in $< T_{\mu\nu} >$ can be seen as correcting the Newton constant since $(4k^{(2)} - 2k^{(3)})/(4k^{(1)} + \frac{k^{(3)}}{2}) = -2$. The spinor field behaves, in this respect, as the minimally-coupled scalar field.

Finally, after tracing and solving the equations of motion, the quantum correction to the Newtonian potential reads $\delta V(r) = -\frac{G^2 M m^2}{3\pi}\frac{\log\frac{r}{r_0}}{r}$ which coincides with the Wilsonian potential, obtained from the RGE for the Newton constant $G(\mu)$.

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REFERENCES

[1] D. A. R. Dalvit and F. D. Mazzitelli, Phys. Rev. D **50**, 1001 (1994).

[2] G.A. Vilkovisky, Proceedings of the Strasbourg Meeting between Physicists and Mathematicians, CERN preprint CERN-TH.6392/92, February 1992.

[3] A. O. Barvinsky and G.A. Vilkovisky, Nucl. Phys. B **282**, 163 (1987); B **333**, 471 (1990).

[4] N.D. Birrell and P.C.W. Davies, *Quantum Fields in Curved Space* (Cambridge Univ. Press, London, 1982).

[5] L. Parker and D.J. Toms, Phys. Rev. D **32**, 1409 (1985).

[6] J.F. Donoghue, Phys. Rev. Lett. **72**, 2996 (1994).

[7] G.A. Vilkovisky, in *Quantum Theory of Gravity*, ed. S.M. Christensen, (Adam Hilger, Bristol, 1984).

[8] J.Z. Simon, Phys. Rev. D**41**, 3720 (1990); D**43**, 308 (1991)

[9] F.D. Mazzitelli, Phys. Rev. D**45**, 2814 (1992)

[10] I.G. Avramidi and A.O. Barvinsky, Phys. Lett. B**159**, 269 (1985)

[11] T. Goldman et al, Phys. Lett. B**281**, 219 (1992)

[12] A. O. Barvinsky et al. , J. Math. Phys. **35**, 3525 (1994); **35**, 3543 (1994)

[13] A. O. Barvinsky et al. , hep-th 9404187, to appear in Nucl. Phys. B.

[14] S. M. Christensen, Phys. Rev. D **17**, 946 (1978).